Assignment 3: AMS 268 (Due Date 3/12)

February 23, 2018

(a) Simulate $\boldsymbol{s}_i = (s_{1i}, s_{2i})'$ uniformly from the domain $[0,1] \times [0,1]$ for $i = 1, ..., 2000$. Simulate

$$y_i = \beta_0 + w_0(\boldsymbol{s}_i) + \epsilon_i, \;\; \epsilon_i \sim N(0, \tau_0^2),$$

for $i = 1, ..., 2000$. Consider, $\tau_0^2 = 0.1$ and simulate $w_0(\boldsymbol{s}_i)$s from a Gaussian process with mean 0 and covariance kernel $\sigma_0^2 \exp(-\phi_0 ||\boldsymbol{s} - \boldsymbol{s}'||)$ with $\sigma_0^2 = 1$ and $\phi_0 = 2$. Take $\beta_0 = 1$ for model fitting.

(i) Use the above model to generate a dataset.

(ii) Run a modified predictive process model with 200 randomly selected knots for the above data by writing your own code. Use exponential covariance kernel for model fitting.

(iii) Comment on the model fit in terms of estimating the model parameters, true spatial surface $w_0(\boldsymbol{s})$.

(iv) Provide in sample mean squared prediction error for the 2000 observations.

(b) Repeat the above exercise for a tapered Gaussian process (or Gaussian process with compactly supported correlation function). Use different choices of the tuning parameter ν to monitor the change in performance.